Coordinate Rotation Corrections in Flux Computations

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Xinhua Zhou, Ph.D., Senior Scientist

Error Sources of Flux Data

- Unrepresentative (designable)
- Measurement uncertainties (uncorrectable)
- **Measurement biases (correctable ☺)**

Major measurement biases

- **Tilt of vertical axis in the sonic anemometer coordinate system away from its counterpart in the natural wind coordinate system.**

  Coordinate rotation correction ☺

- **Frequency loss due to the gradual response and line/volume averaging to measured variables.**

  Frequency correction ☺

- **Air density fluctuations due to heat and water transfer into/out the measured air flows**

  WPL correction (Density effect corrections) ☺

  Webb, **P**earman, and **L**euning (1980)

- **Use of sonic temperature for sensible heat flux**

  SND correction (Moisture correction of sonic temperature flux for sensible heat flux) ☺

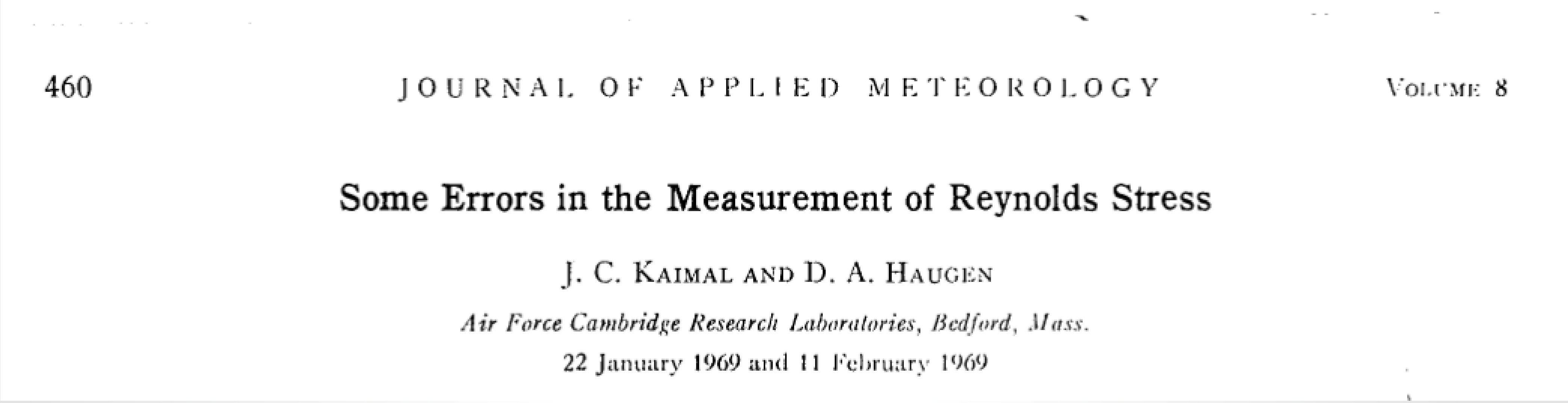
  Schotanus, **N**ieuwstadt, **D**ebruin (1983)

Flux

CO2 $$F_{CO_2} = \rho_d \overline{w' \chi_{CO_2}}$$

H2O $$F_{H_2O} = \rho_d \overline{w' \chi_{H_2O}}$$

Momentum $$\tau = \rho \left(\overline{w'u'}^2 + \overline{w'v'}^2 \right)^{\frac{1}{2}}$$



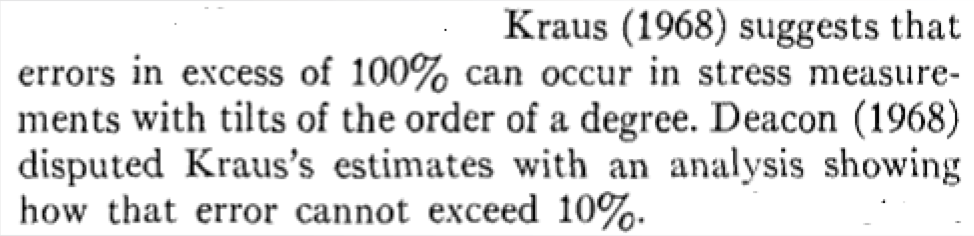
460 JOURNAL OF APPLIED METEOROLOGY VOLUME 8

Some Errors in the Measurement of Reynolds Stress

J. C. KAIMAL AND D. A. HAUGEN

Air Force Cambridge Research Laboratories, Bedford, Mass.

22 January 1969 and 11 February 1969



Kraus (1968) suggests that errors in excess of 100% can occur in stress measurements with tilts of the order of a degree. Deacon (1968) disputed Kraus's estimates with an analysis showing how that error cannot exceed 10%.

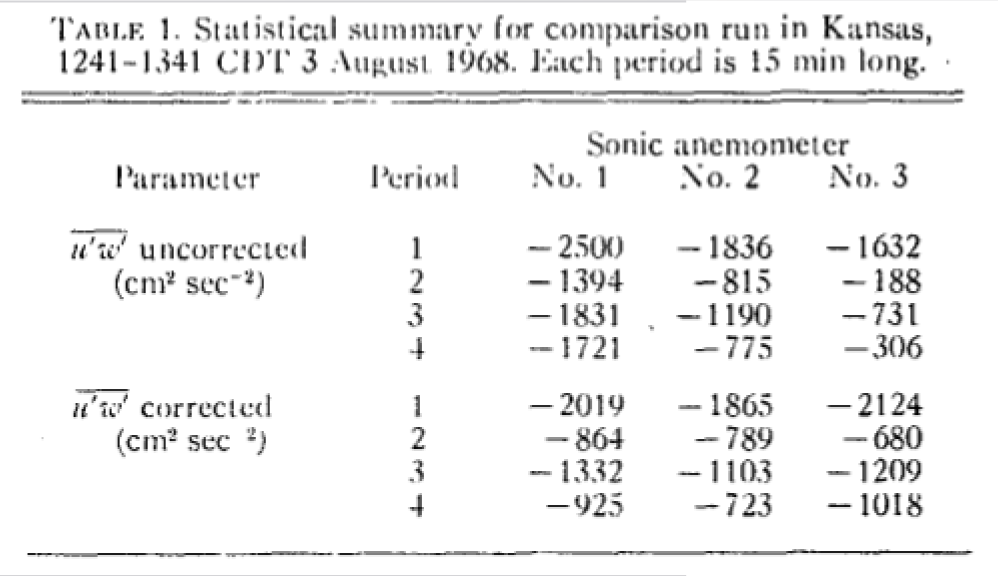
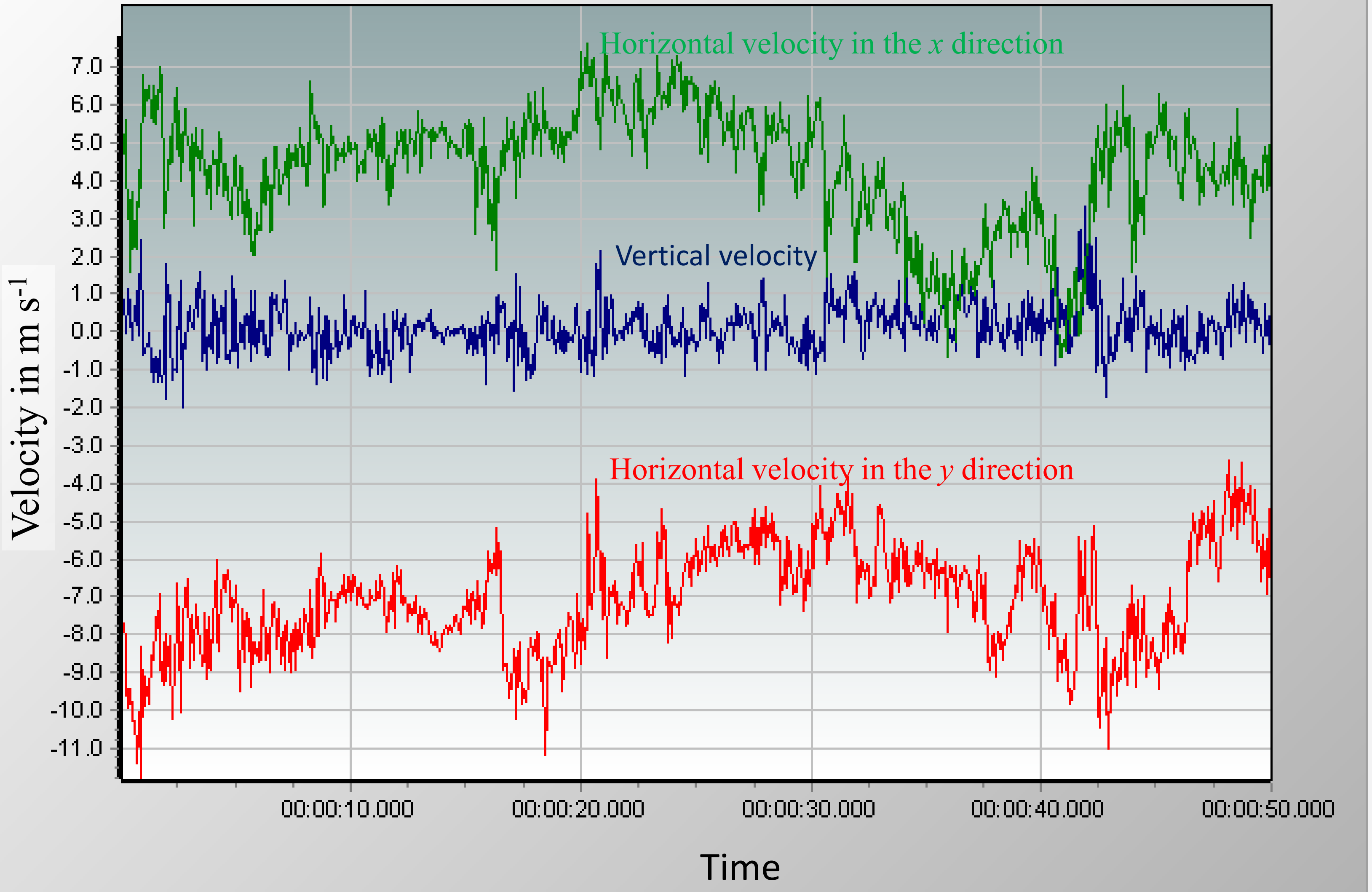


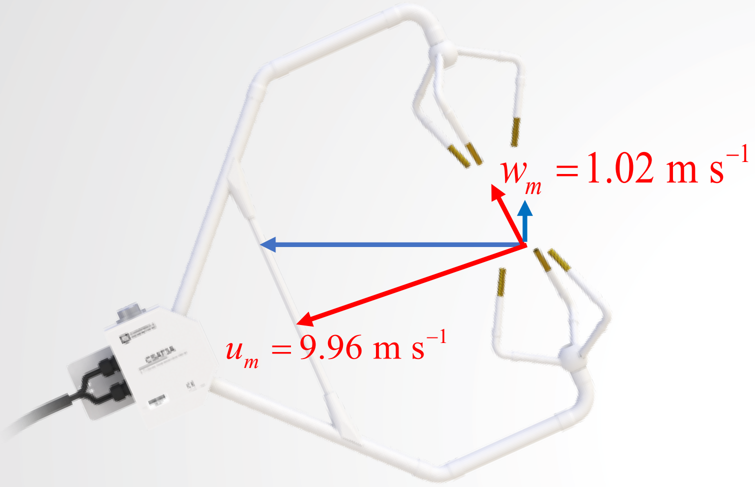
TABLE 1. Statistical summary for comparison run in Kansas, 1241–1341 CDT 3 August 1968. Each period is 15 min long.

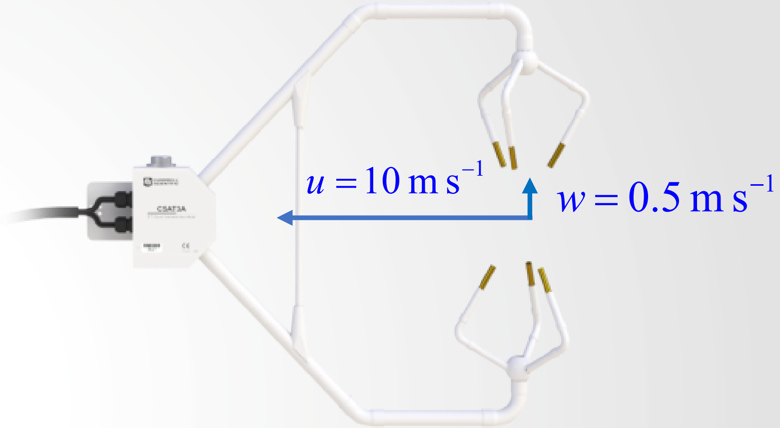
Parameter	Period	Sonic anemometer No. 1	Sonic anemometer No. 2	Sonic anemometer No. 3
$\overline{u'w'}$ uncorrected $(\text{cm}^2\ \text{sec}^{-2})$	1	−2500	−1836	−1632
	2	−1394	−815	−188
	3	−1831	−1190	−731
	4	−1721	−775	−306
$\overline{u'w'}$ corrected $(\text{cm}^2\ \text{sec}^{-2})$	1	−2019	−1865	−2124
	2	−864	−789	−680
	3	−1332	−1103	−1209
	4	−925	−723	−1018

Wind velocities measured at 20 Hz
in a coordinate system of 3D sonic anemometer



Tilt angle = 3°

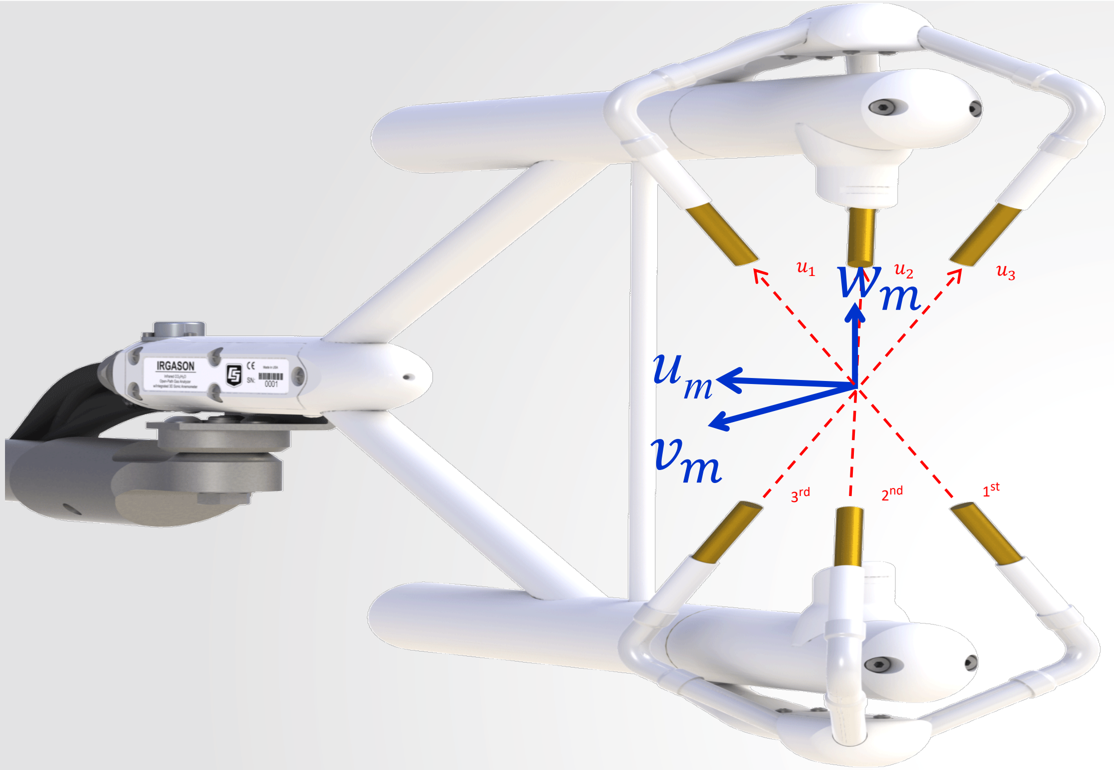




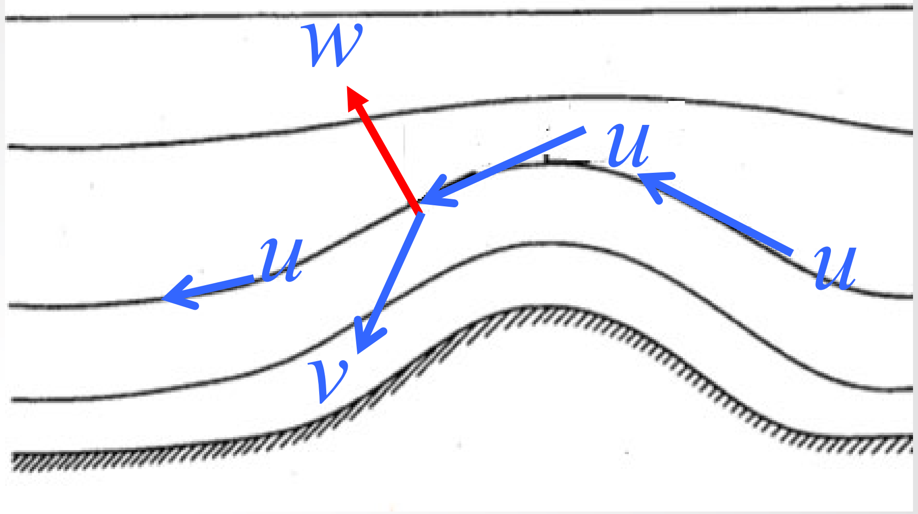
$$w_m = 10\cos 87^o + 0.5\cos 3^o = 1.02 \,\text{m s}^{-1} \qquad \text{relative error} = 100\%$$

$$u_m = 10\cos 3^o + 0.5\cos 93^o = 9.96 \,\text{m s}^{-1} \qquad \text{relative error} = 0.4\%$$

Installation tilt

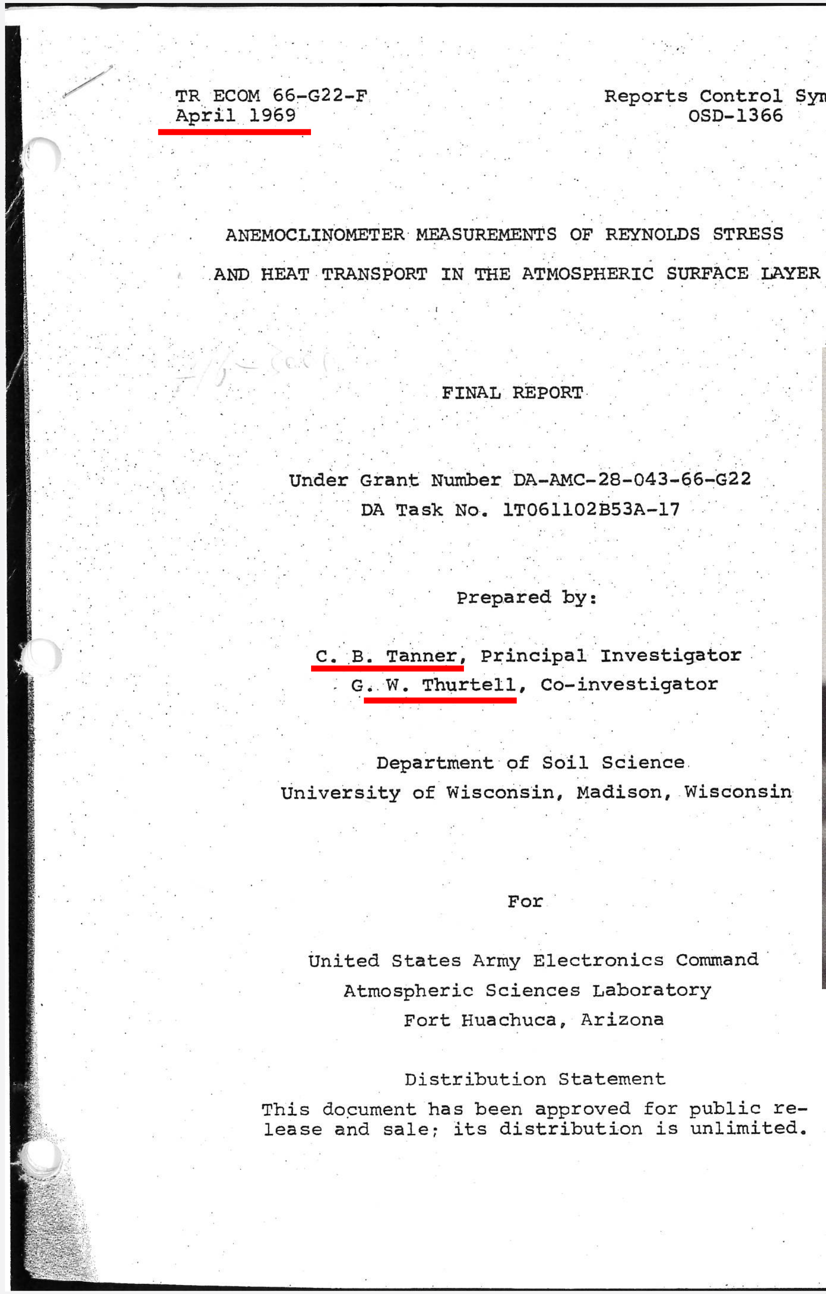


CSAT right-handed
coordinate system



Coordinate rotation corrections

Express the measured fluxes including 3D wind velocities in a sonic anemometer coordinate system into the natural wind coordinate system.



TR ECOM 66-G22-F
April 1969

Reports Control Symbol
OSD-1366

ANEMOCLINOMETER MEASUREMENTS OF REYNOLDS STRESS
AND HEAT TRANSPORT IN THE ATMOSPHERIC SURFACE LAYER

FINAL REPORT

Under Grant Number DA-AMC-28-043-66-G22
DA Task No. 1T061102B53A-17

Prepared by:

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University of Wisconsin, Madison, Wisconsin

For

United States Army Electronics Command
Atmospheric Sciences Laboratory
Fort Huachuca, Arizona

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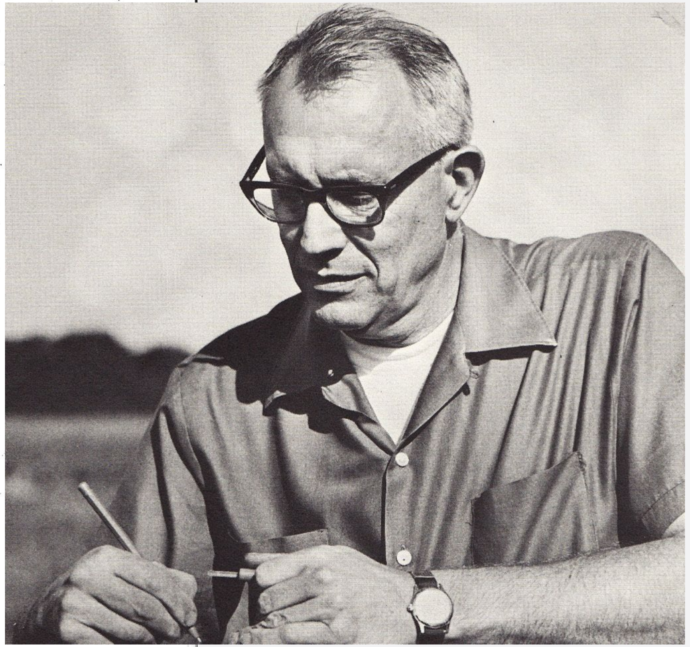
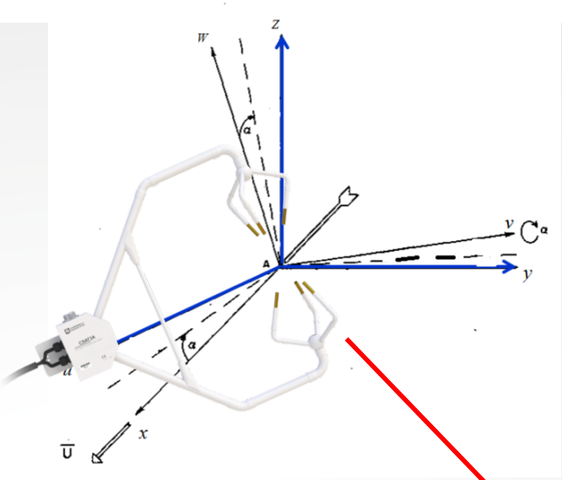
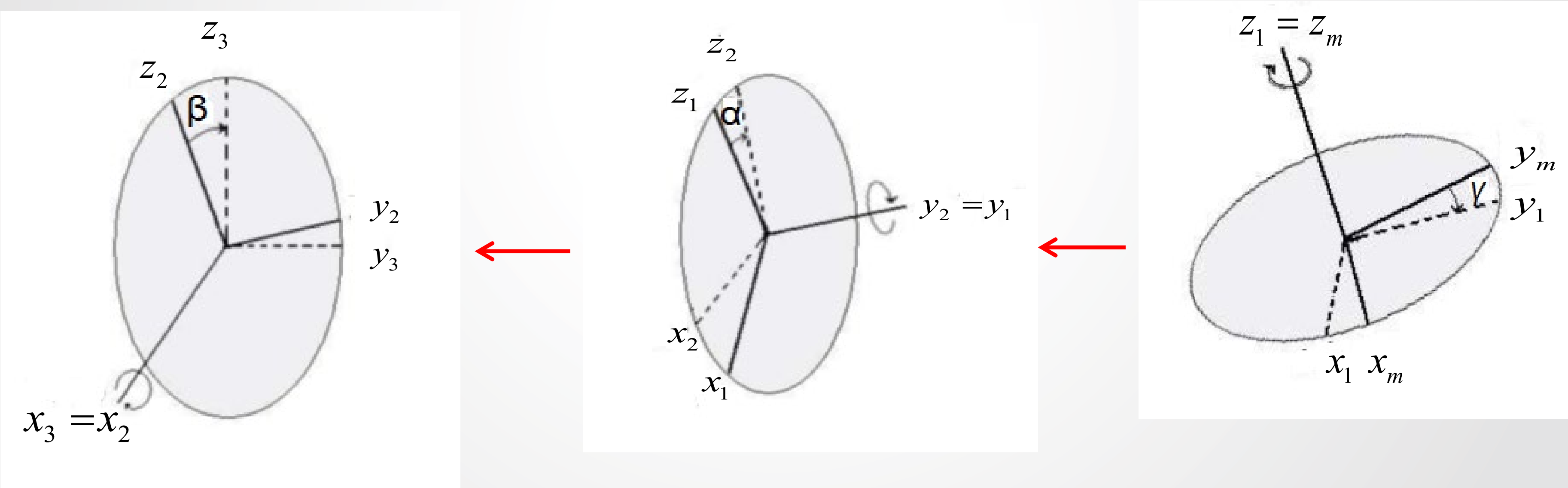


Sonic to natural wind coordinate system

(Tanner and Thurtell, 1969)



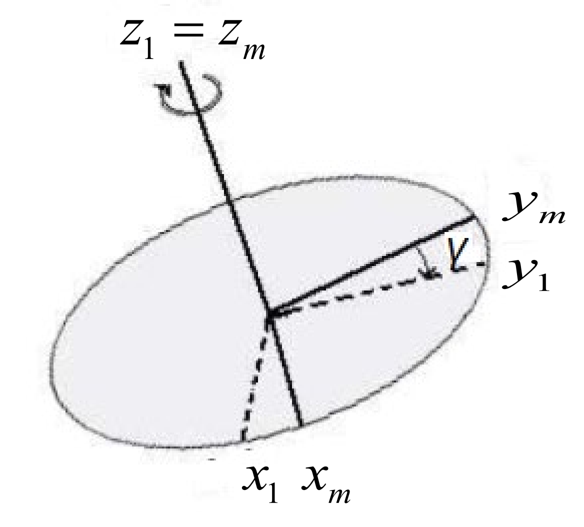


Tanner, CB, GW Thurtell. 1969. Anemoclinometer measurements of Reynolds stress and heat transport in the atmospheric surface layer, US Army Electronics Command, Atmospheric Sciences Laboratory, TR ECOM 66-G22-F, R1-R10.

1st Rotation around z_m-axis aligns to the mainstream wind direction at a temporal scale of averaging data.

$$\gamma = \arctan\left(\frac{\overline{v}_m}{\overline{u}_m}\right)$$

$$\begin{bmatrix} \overline{u}_1 \\ \overline{v}_1 \\ \overline{w}_1 \end{bmatrix} = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \overline{u}_m \\ \overline{v}_m \\ \overline{w}_m \end{bmatrix}$$

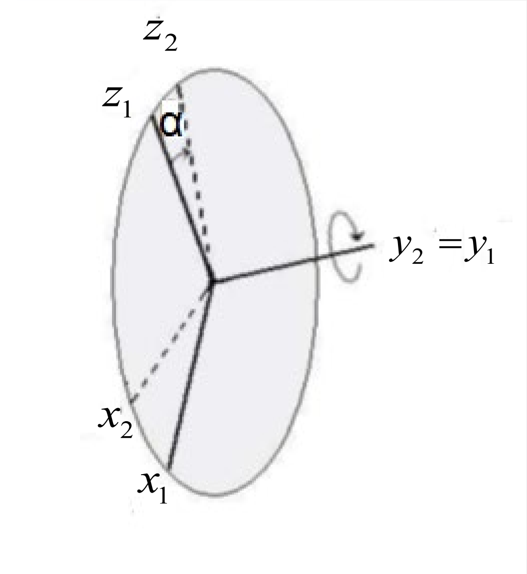


2nd Rotation around new y_1-axis nullifies mean vertical wind

$$\alpha = -\arctan\frac{\overline{w}_1}{\overline{u}_1}$$

$$= -\arctan\frac{\overline{w}_m}{\overline{u}_m\cos\gamma + \overline{v}_m\sin\gamma}$$

$$\begin{bmatrix}\overline{u}_2\\ \overline{v}_2\\ \overline{w}_2\end{bmatrix} = \begin{bmatrix}\cos\alpha & 0 & \sin\alpha\\ 0 & 1 & 0\\ -\sin\alpha & 0 & \sin\alpha\end{bmatrix}\begin{bmatrix}\overline{u}_1\\ \overline{v}_1\\ \overline{w}_1\end{bmatrix}$$



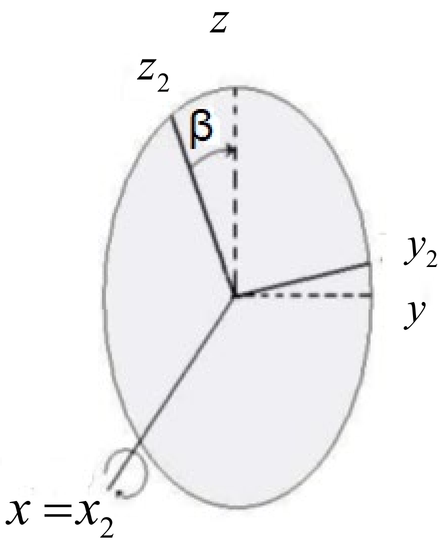
3rd Rotation around new x_2-axis to IDEALLY express the measured 3D wind means to the natural wind coordinate system.

Assuming $\overline{v_m^{'} w_m^{'}} = 0$,

$$\beta = \frac{1}{2} \arctan\left(2 \frac{\overline{v_2^{'} w_2^{'}}}{\overline{v_2^{'}} - \overline{w_2^{'}}} \right)$$

Rotation around x_2-axis.

$$\begin{bmatrix} \overline{u} \\ \overline{v} \\ \overline{w} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta \\ 0 & \sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} \overline{u}_2 \\ \overline{v}_2 \\ \overline{w}_2 \end{bmatrix}$$

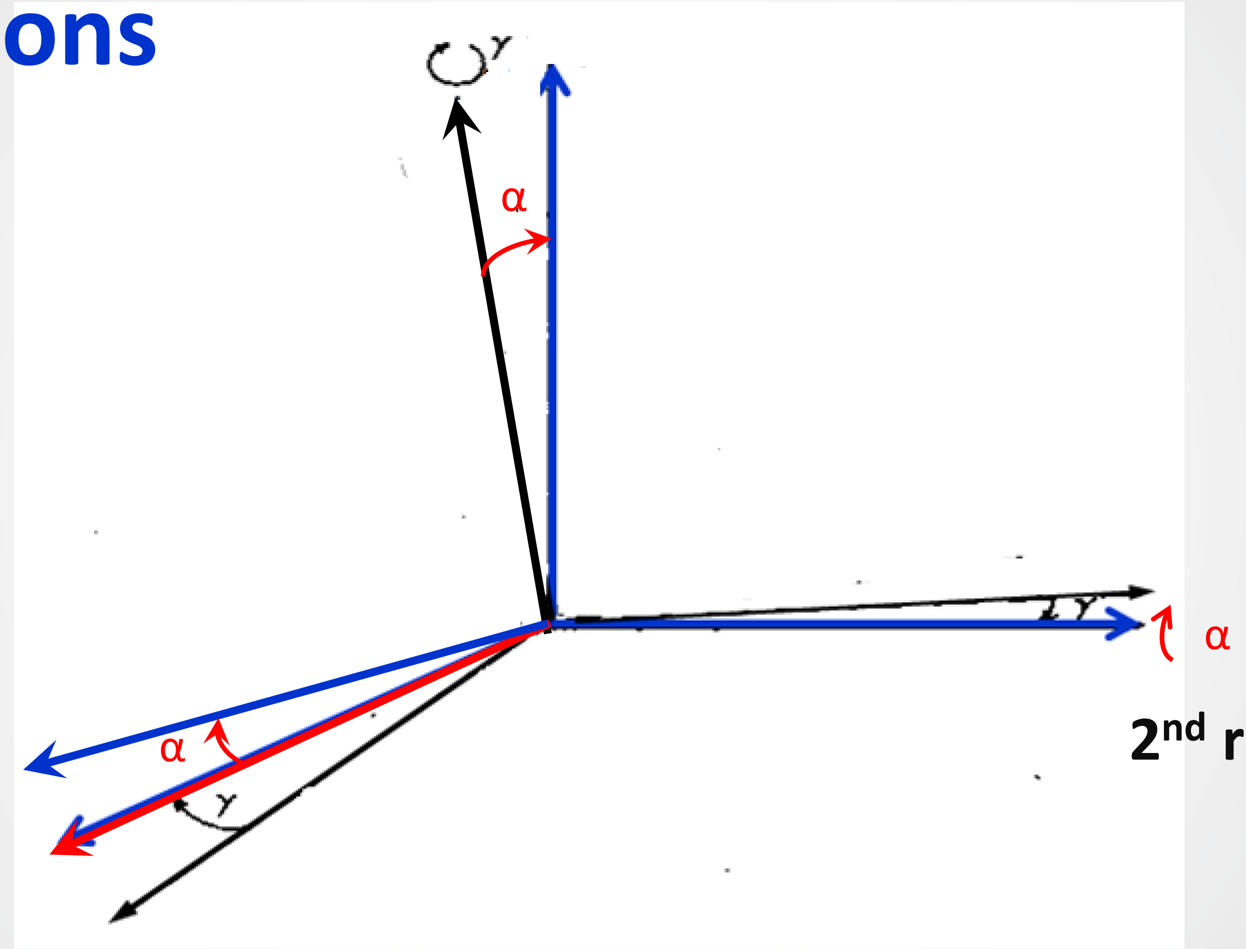


The assumption to derive β is invalid. The 3rd rotation is not recommended any more (Finnigan, 2004)

Finnigan, J. J.: 2004, 'A re-evaluation of long-term flux measurement techniques Part II: Coordinates systems', *Boundary-Layer Meteorology*. **113**, 1-41

Summary

Double rotations



1st rotation

2nd rotation

$$\begin{bmatrix} u_2 \\ v_2 \\ w_2 \end{bmatrix} = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_m \\ v_m \\ w_m \end{bmatrix}$$

Double coordinate rotations for mean wind speeds

$$\begin{bmatrix} \overline{u}_2 \\ \overline{v}_2 \\ \overline{w}_2 \end{bmatrix} = \begin{bmatrix} \cos\alpha\cos\gamma & \cos\alpha\sin\gamma & -\sin\alpha \\ -\sin\gamma & \cos\gamma & 0 \\ \sin\alpha\cos\gamma & \sin\alpha\sin\gamma & \cos\alpha \end{bmatrix} \begin{bmatrix} \overline{u}_m \\ \overline{v}_m \\ \overline{w}_m \end{bmatrix} = \mathbf{R}_2 \begin{bmatrix} \overline{u}_m \\ \overline{v}_m \\ \overline{w}_m \end{bmatrix}$$

Double coordinate rotations for fluctuations in wind speeds

$$\begin{bmatrix} u_2' \\ v_2' \\ w_2' \end{bmatrix} = \mathbf{R_2} \begin{bmatrix} u_m' \\ v_m' \\ w_m' \end{bmatrix}$$

Double coordinate rotations for momentum covariance terms

$$\begin{bmatrix} u_2' \\ v_2' \\ w_2' \end{bmatrix} \begin{bmatrix} u_2' & v_2' & w_2' \end{bmatrix} = \mathbf{R_2} \begin{bmatrix} u_m' \\ v_m' \\ w_m' \end{bmatrix} \begin{bmatrix} u_m' & v_m' & w_m' \end{bmatrix} \mathbf{R_2^T}$$

$$\begin{bmatrix} \overline{u_2'^2} & \overline{u_2' v_2'} & \overline{u_2' w_2'} \\ \overline{u_2' v_2'} & \overline{v_2'^2} & \overline{v_2' w_2'} \\ \overline{u_2' w_2'} & \overline{v_2' w_2'} & \overline{w_2'^2} \end{bmatrix} = \mathbf{R_2} \begin{bmatrix} \overline{u_m'^2} & \overline{u_m' v_m'} & \overline{u_m' w_m'} \\ \overline{u_m' v_m'} & \overline{v_m'^2} & \overline{v_m' w_m'} \\ \overline{u_m' w_m'} & \overline{v_m' w_m'} & \overline{w_m'^2} \end{bmatrix} \mathbf{R_2^T}$$

Double coordinate rotations for covariance of CO$_2$ density with momentum variables

$$\rho'_{CO_2}\begin{bmatrix}u'_2\\ v'_2\\ w'_2\end{bmatrix}=\mathbf{R_2}\,\rho'_{CO_2}\begin{bmatrix}u'_m\\ v'_m\\ w'_m\end{bmatrix}$$

$$\begin{bmatrix}\overline{\rho'_{CO_2}u'_2}\\ \overline{\rho'_{CO_2}v'_2}\\ \overline{\rho'_{CO_2}w'_2}\end{bmatrix}=\mathbf{R_2}\begin{bmatrix}\overline{\rho'_{CO_2}u'_m}\\ \overline{\rho'_{CO_2}v'_m}\\ \overline{\rho'_{CO_2}w'_m}\end{bmatrix}=\begin{bmatrix}\cos\alpha\left(\overline{\rho'_{CO_2}u'_m}\cos\gamma+\overline{\rho'_{CO_2}v'_m}\sin\gamma\right)-\overline{\rho'_{CO_2}w'_m}\sin\alpha\\ -\overline{\rho'_{CO_2}u'_m}\sin\gamma+\overline{\rho'_{CO_2}v'_m}\cos\gamma\\ \sin\alpha\left(\overline{\rho'_{CO_2}u'_m}\cos\gamma+\overline{\rho'_{CO_2}v'_m}\sin\gamma\right)+\overline{\rho'_{CO_2}w'_m}\cos\alpha\end{bmatrix}$$

Applicable for CO$_2$ mixing ratio *(χ_{CO_2})* with momentum variables *by replacing with ρ_{CO_2} with χ_{CO_2}* .

Double coordinate rotations for covariance of H_2O density with momentum variables

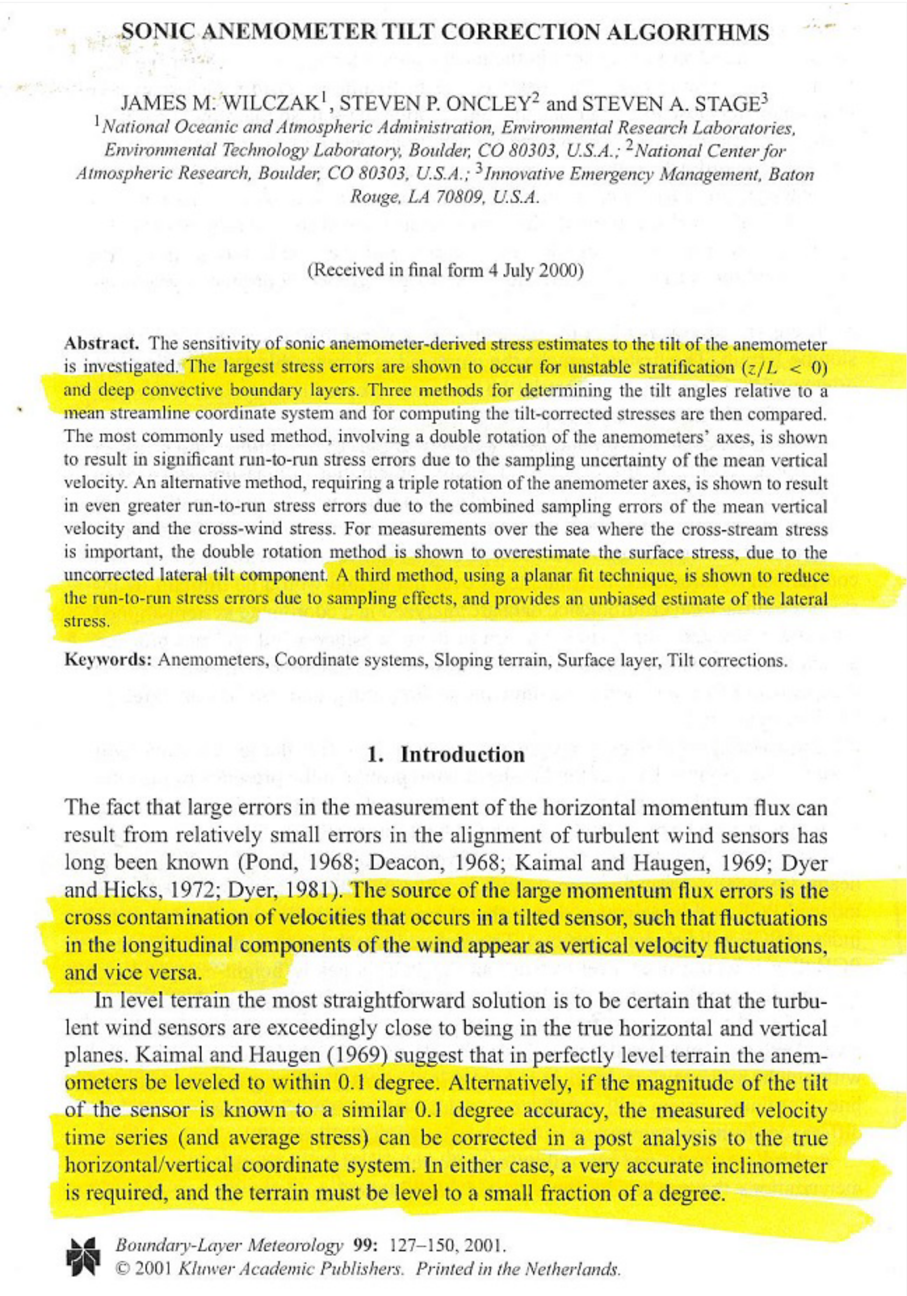
$$\rho'_{H_2O}\begin{bmatrix} u'_2 \\ v'_2 \\ w'_2 \end{bmatrix} = \mathbf{R}_2\,\rho'_{H_2O}\begin{bmatrix} u'_m \\ v'_m \\ w'_m \end{bmatrix}$$

$$\begin{bmatrix} \overline{\rho'_{H_2O}u'_2} \\ \overline{\rho'_{H_2O}v'_2} \\ \overline{\rho'_{H_2O}w'_2} \end{bmatrix} = \mathbf{R}_2\begin{bmatrix} \overline{\rho'_{H_2O}u'_m} \\ \overline{\rho'_{H_2O}v'_m} \\ \overline{\rho'_{H_2O}w'_m} \end{bmatrix} = \begin{bmatrix} \cos\alpha\left(\overline{\rho'_{H_2O}u'_m}\cos\gamma + \overline{\rho'_{H_2O}v'_m}\sin\gamma\right) - \overline{\rho'_{H_2O}w'_m}\sin\alpha \\ -\overline{\rho'_{H_2O}u'_m}\sin\gamma + \overline{\rho'_{H_2O}v'_m}\cos\gamma \\ \sin\alpha\left(\overline{\rho'_{H_2O}u'_m}\cos\gamma + \overline{\rho'_{H_2O}v'_m}\sin\gamma\right) + \overline{\rho'_{H_2O}w'_m}\cos\alpha \end{bmatrix}$$

Applicable for H_2O mixing ratio (χ_{H_2O}) with momentum variables *by replacing with* ρ_{CO_2} *with* χ_{CO_2} .

Planner fit rotations

(Wilczak et al., 2001)



SONIC ANEMOMETER TILT CORRECTION ALGORITHMS

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[1]*National Oceanic and Atmospheric Administration, Environmental Research Laboratories, Environmental Technology Laboratory, Boulder, CO 80303, U.S.A.;* [2]*National Center for Atmospheric Research, Boulder, CO 80303, U.S.A.;* [3]*Innovative Emergency Management, Baton Rouge, LA 70809, U.S.A.*

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Abstract. The sensitivity of sonic anemometer-derived stress estimates to the tilt of the anemometer is investigated. The largest stress errors are shown to occur for unstable stratification ($z/L < 0$) and deep convective boundary layers. Three methods for determining the tilt angles relative to a mean streamline coordinate system and for computing the tilt-corrected stresses are then compared. The most commonly used method, involving a double rotation of the anemometers' axes, is shown to result in significant run-to-run stress errors due to the sampling uncertainty of the mean vertical velocity. An alternative method, requiring a triple rotation of the anemometer axes, is shown to result in even greater run-to-run stress errors due to the combined sampling errors of the mean vertical velocity and the cross-wind stress. For measurements over the sea where the cross-stream stress is important, the double rotation method is shown to overestimate the surface stress, due to the uncorrected lateral tilt component. A third method, using a planar fit technique, is shown to reduce the run-to-run stress errors due to sampling effects, and provides an unbiased estimate of the lateral stress.

Keywords: Anemometers, Coordinate systems, Sloping terrain, Surface layer, Tilt corrections.

1. Introduction

The fact that large errors in the measurement of the horizontal momentum flux can result from relatively small errors in the alignment of turbulent wind sensors has long been known (Pond, 1968; Deacon, 1968; Kaimal and Haugen, 1969; Dyer and Hicks, 1972; Dyer, 1981). The source of the large momentum flux errors is the cross contamination of velocities that occurs in a tilted sensor, such that fluctuations in the longitudinal components of the wind appear as vertical velocity fluctuations, and vice versa.

In level terrain the most straightforward solution is to be certain that the turbulent wind sensors are exceedingly close to being in the true horizontal and vertical planes. Kaimal and Haugen (1969) suggest that in perfectly level terrain the anemometers be leveled to within 0.1 degree. Alternatively, if the magnitude of the tilt of the sensor is known to a similar 0.1 degree accuracy, the measured velocity time series (and average stress) can be corrected in a post analysis to the true horizontal/vertical coordinate system. In either case, a very accurate inclinometer is required, and the terrain must be level to a small fraction of a degree.

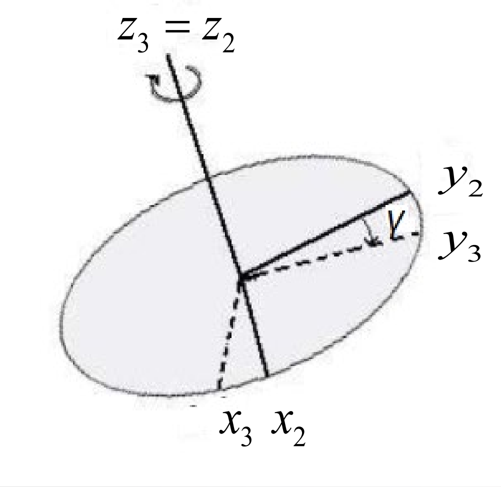
Boundary-Layer Meteorology **99:** 127–150, 2001.

Planner fit rotations

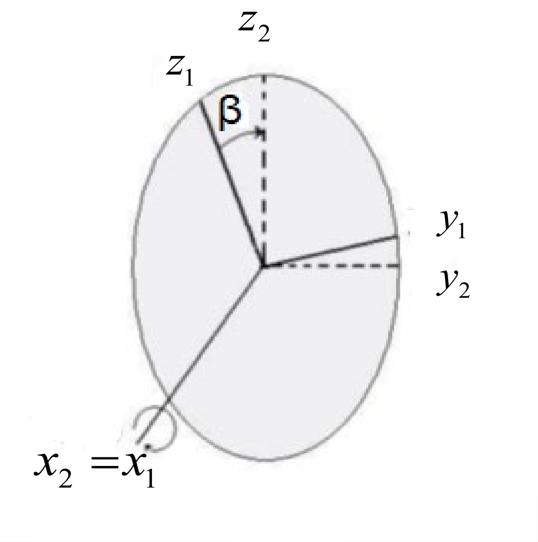
(Wilczak et al., 2001)

3rd rotation

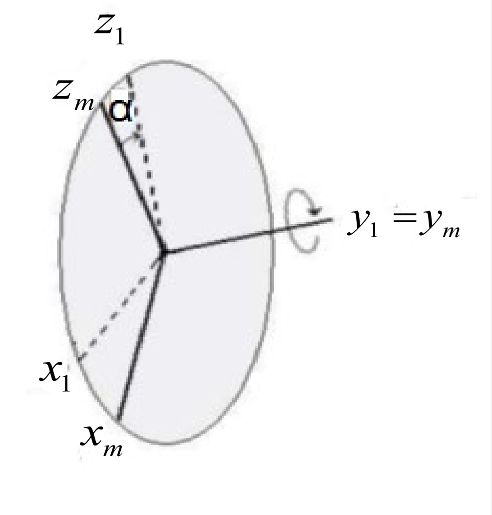
(Unnecessary if only for flux)



2nd rotation

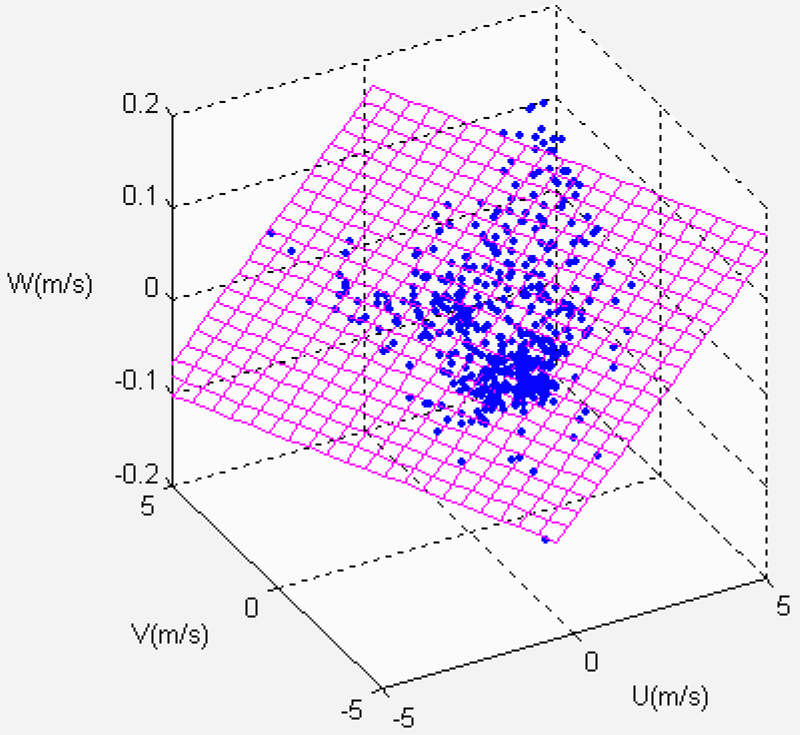


1st rotation



Wilczak, JM, SP Oncley, SA Stage. 2001. Sonic Anemometer tilt correction algorithm. Boundary-Layer Meteorol. 99: 127–150.

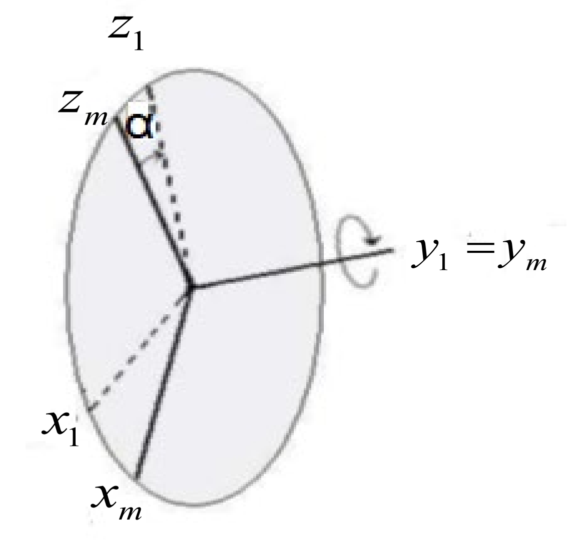
Rotation angle computations



Coordinate system rotated about this plane

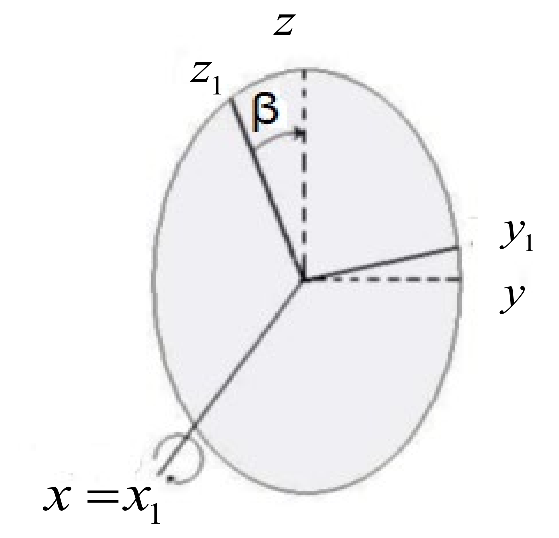
- Computation of rotation angles (α, β, and γ) need two- to three- week data
- Algorithms to compute the rotation angles are long, which are ignored here.

1st Rotation around y_m-axis



$$\begin{bmatrix} \bar{u}_1 \\ \bar{v}_1 \\ \bar{w}_1 \end{bmatrix} = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \sin\alpha \end{bmatrix} \begin{bmatrix} \bar{u}_m \\ \bar{v}_m \\ \bar{w}_m \end{bmatrix}$$

2nd Rotation around new x_1-axis.



$$\begin{bmatrix} \bar{u} \\ \bar{v} \\ \bar{w} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta \\ 0 & \sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} \bar{u}_1 \\ \bar{v}_1 \\ \bar{w}_1 \end{bmatrix}$$

Planar fit matrixes

$$\begin{bmatrix} \overline{u} \\ \overline{v} \\ \overline{w} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\beta & -\sin\beta \\ 0 & \sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} \overline{u}_m \\ \overline{v}_m \\ \overline{w}_m \end{bmatrix}$$

$$\begin{bmatrix} \overline{u} \\ \overline{v} \\ \overline{w} \end{bmatrix} = \mathrm{R}_{\mathrm{p}} \begin{bmatrix} \overline{u}_m \\ \overline{v}_m \\ \overline{w}_m \end{bmatrix}$$

Planar fit coordinate rotations for momentum covariance terms

$$\begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} \begin{bmatrix} u' & v' & w' \end{bmatrix} = \mathbf{R_P} \begin{bmatrix} u'_m \\ v'_m \\ w'_m \end{bmatrix} \begin{bmatrix} u'_m & v'_m & w'_m \end{bmatrix} \mathbf{R_P^T}$$

$$\begin{bmatrix} \overline{u'^2} & \overline{u'v'} & \overline{u'w'} \\ \overline{u'v'} & \overline{v'^2} & \overline{v'w'} \\ \overline{u'w'} & \overline{v'w'} & \overline{w'^2} \end{bmatrix} = \mathbf{R_2} \begin{bmatrix} \overline{u'^2_m} & \overline{u'_m v'_m} & \overline{u'_m w'_m} \\ \overline{u'_m v'_m} & \overline{v'^2_m} & \overline{v'_m w'_m} \\ \overline{u'_m w'_m} & \overline{v'_m w'_m} & \overline{w'^2_m} \end{bmatrix} \mathbf{R_2^T}$$

Planar fit coordinate rotations for covariance of CO$_2$ density with momentum variables

$$\rho'_{CO_2}\begin{bmatrix}u'\\ v'\\ w'\end{bmatrix} = \mathbf{R_P}\,\rho'_{CO_2}\begin{bmatrix}u'_m\\ v'_m\\ w'_m\end{bmatrix} \longrightarrow \begin{bmatrix}\overline{\rho'_{CO_2}u'}\\ \overline{\rho'_{CO_2}v'}\\ \overline{\rho'_{CO_2}w'}\end{bmatrix} = \mathbf{R_P}\begin{bmatrix}\overline{\rho'_{CO_2}u'_m}\\ \overline{\rho'_{CO_2}v'_m}\\ \overline{\rho'_{CO_2}w'_m}\end{bmatrix}$$

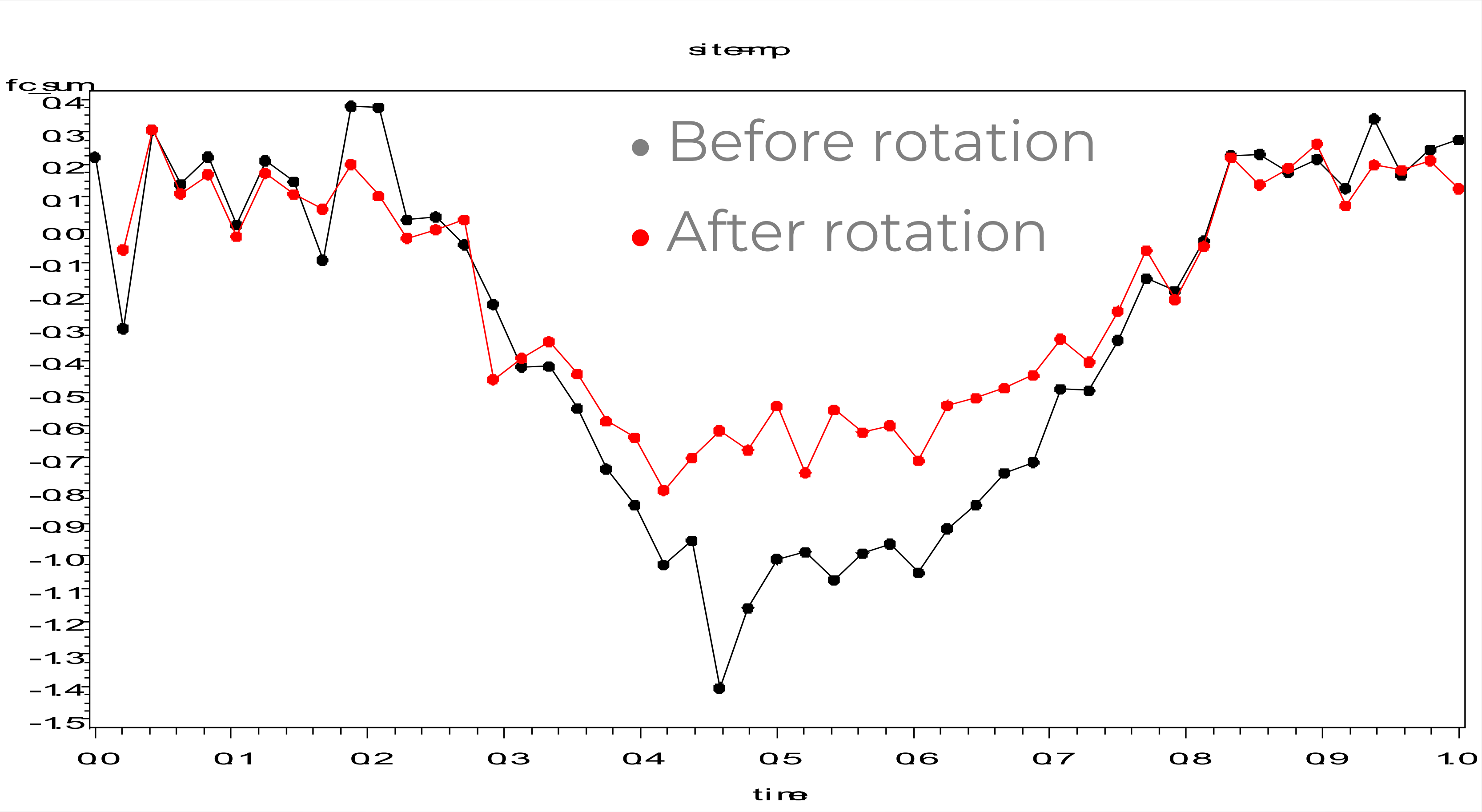
Applicable for CO$_2$ mixing ratio (χ_{CO_2}) with momentum variables *by replacing with ρ_{CO_2} with χ_{CO_2}* .

Planar fit coordinate rotations for covariance of H_2O density with momentum variables

$$\rho'_{H_2O}\begin{bmatrix}u'\\ v'\\ w'\end{bmatrix} = \mathbf{R_P}\,\rho'_{H_2O}\begin{bmatrix}u'_m\\ v'_m\\ w'_m\end{bmatrix} \longrightarrow \begin{bmatrix}\overline{\rho'_{H_2O}u'_2}\\ \overline{\rho'_{H_2O}v'_2}\\ \overline{\rho'_{H_2O}w'_2}\end{bmatrix} = \mathbf{R_P}\begin{bmatrix}\overline{\rho'_{H_2O}u'_m}\\ \overline{\rho'_{H_2O}v'_m}\\ \overline{\rho'_{H_2O}w'_m}\end{bmatrix}$$

Applicable for H_2O mixing ratio (χ_{H_2O}) with momentum variables *by replacing with ρ_{CO_2} with χ_{CO_2}* .

CO_2 Flux


sitemp
fcsum
0.4
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1.0
-1.1
-1.2
-1.3
-1.4
-1.5
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
time
● Before rotation
● After rotation

Major References

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