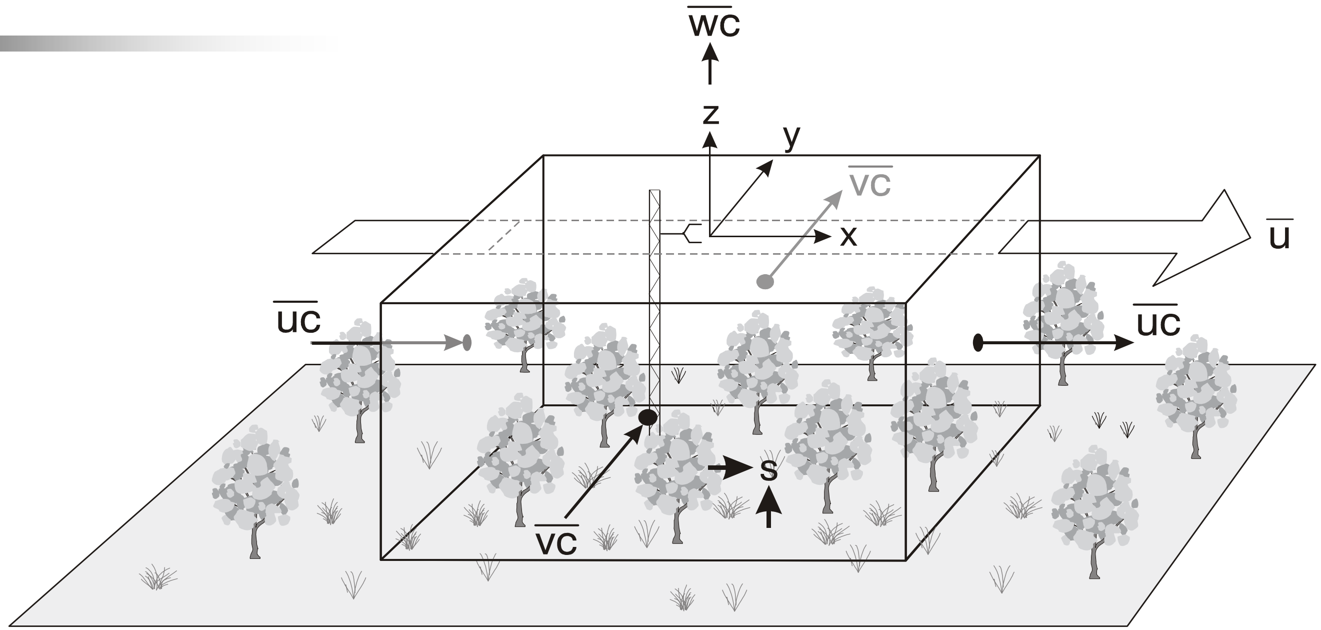
Lecture 3:
Eddy flux measurement theory

- Conservation of mass equation for a control volume
- Webb, Pearman & Leuning theory
- Open-path instruments
- Closed-path instruments
- Summary

Control volume over surface around tower



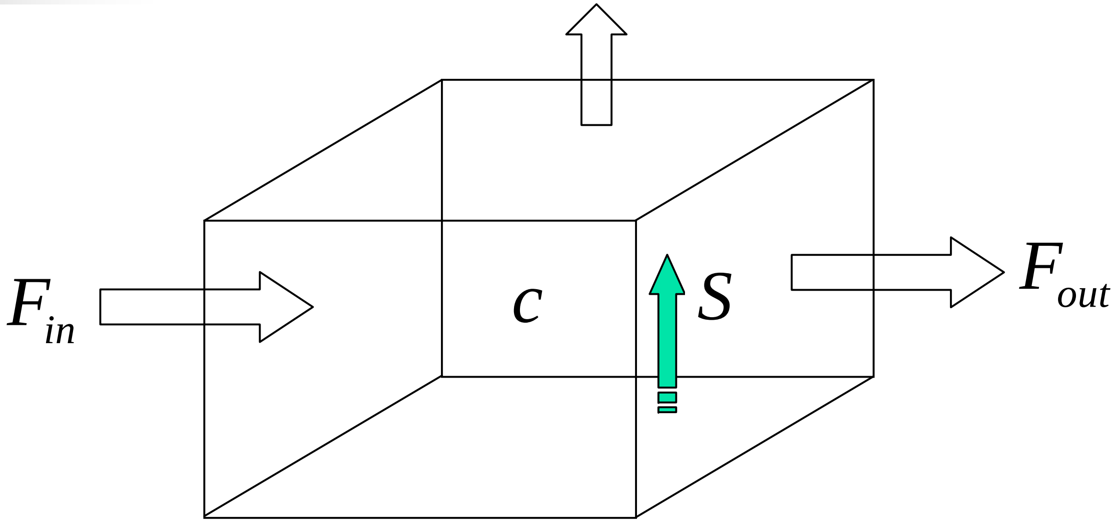
We want sum of CO_2 & H_2O fluxes from soil and plant surfaces within CV

We measure eddy flux at top of tower $\overline{wc}$

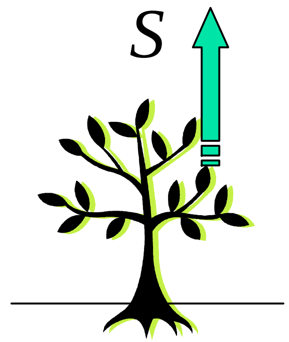
- When are they equal?
- What assumptions, simplifications are needed?

Mass conservation in small volume *dV*

Leuning (2004, Ch 6)



$$\frac{\partial c}{\partial t} + \nabla.\mathbf{F} = S_d + S_c$$



Use vector notation ($\nabla.\mathbf{F} = \partial F_x / \partial x + \partial F_y / \partial y + \partial F_z / \partial z$)

Mass conservation in small volume *dV*

$$\frac{\partial(c_d + c_c)}{\partial t} + \nabla.(\mathbf{F}_d + \mathbf{F}_c) = \cancelto{0}{S_d} + S_c$$

- Consider dry air and quantity *c* separately

$$\frac{\partial c_d}{\partial t} + \nabla.\mathbf{u}c_d = 0$$ No sources/sinks of dry air in CV

$$\frac{\partial c_c}{\partial t} + \nabla.\mathbf{u}c_c = S_c$$ Sources/sinks of *c* in CV

Show that $\overline{S}_c = \overline{\frac{\partial c_c}{\partial t}} + \nabla.\overline{\mathbf{u}c_c}$ equals

$$\boxed{\overline{S}_c = \overline{c}_d \overline{\frac{\partial \chi_c}{\partial t}} + \overline{\mathbf{u}c_d}.\nabla\overline{\chi}_c + \nabla.(\overline{c}_d \overline{\mathbf{u}^{'}\chi_c^{'}})}$$

$\overline{c}_d$ = concentration of dry air

$\overline{\chi}_c$ = mixing ratio of quantity *c* in dry air

$\mathbf{u} = (u, v, w)$ = velocity vector

$\overline{S}_c$ = source flux density

Reynolds decomposition $\chi_c = \overline{\chi} + \chi_c^{'},\ u = \overline{u} + u^{'}$

Overbar = time average $\overline{\chi} = (1/\Delta t)\int_t^{t+\Delta t} \chi dt$

Conservation equation for quantity *c*

$$\overline{\frac{\partial c_c}{\partial t}} + \nabla.\overline{\mathbf{u}c_c} = \overline{S}_c$$

Use mixing ratio for *c* and Reynolds decomposition

$$c_c = c_d \chi_c = (\overline{c}_d + c_d^{'})(\overline{\chi}_c + \chi_c^{'}) \qquad \mathbf{u} = (\overline{\mathbf{u}} + \mathbf{u}^{'})$$

Mixing ratio

Mean

Fluctuation

$$\overline{\frac{\partial(\overline{c}_d + c_d^{'})(\overline{\chi}_c + \chi_c^{'})}{\partial t}} + \nabla.\overline{[(\overline{\mathbf{u}} + \mathbf{u}^{'})(\overline{c}_d + c_d^{'})(\overline{\chi}_c + \chi_c^{'})]} = \overline{S}_c$$

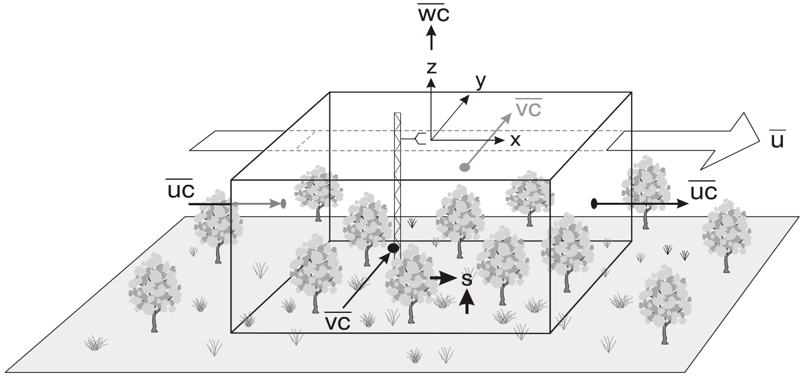
After some algebra- see Leuning (2004)

$$\bar{c}_d \frac{\overline{\partial \chi_c}}{\partial t} + \bar{\chi}_c \left[\cancelto{0}{\frac{\overline{\partial c_d}}{\partial t} + \nabla . \overline{\mathbf{u} c_d}} \right] + \overline{\mathbf{u} c_d} . \nabla \bar{\chi}_c + \nabla . (\bar{c}_d \overline{\mathbf{u}' \chi_c'}) = \bar{S}_c$$

$$\boxed{\bar{c}_d \frac{\overline{\partial \chi_c}}{\partial t} + \overline{\mathbf{u} c_d} . \nabla \bar{\chi}_c + \nabla . (\bar{c}_d \overline{\mathbf{u}' \chi_c'}) = \bar{S}_c}$$

Δ storage | advection | eddy flux divergence | Source flux density

Mass conservation in CV around flux tower



$$\overline{F_0} = \frac{1}{L^2}\int_0^L\int_0^L\int_0^h \overline{c_d}\,\overline{\frac{\partial \chi_c}{\partial t}}\,dxdydz$$

$$+\frac{1}{L^2}\int_0^L\int_0^L\int_0^h \left[\overline{uc_d}\frac{\partial \overline{\chi_c}}{\partial x} + \overline{vc_d}\frac{\partial \overline{\chi_c}}{\partial y} + \overline{wc_d}\frac{\partial \overline{\chi_c}}{\partial z}\right] dxdydz$$

$$+\frac{1}{L^2}\int_0^L\int_0^L\int_0^h \left[\frac{\partial \overline{c_d}\,\overline{u'\chi_c'}}{\partial x} + \frac{\partial \overline{c_d}\,\overline{v'\chi_c'}}{\partial y} + \frac{\partial \overline{c_d}\,\overline{w'\chi_c'}}{\partial z}\right] dxdydz$$

Note spatial averaging

Coordinate system

- Have used rectangular Cartesian coordinates
- Can rarely measure all components of mass balance. To maximize information at tower choose site and coordinate system to ensure:

$$\overline{vc_d} = \overline{wc_d} = 0$$

Coordinate rotation (see later)

$$\frac{\partial \overline{\chi_c}}{\partial x}, \frac{\partial \overline{\chi_c}}{\partial y} \to 0$$

Horizontal homogeneity – no advection

$$\frac{\partial \overline{w'\chi_c'}}{\partial z} \; ? \; \frac{\partial \overline{u'\chi_c'}}{\partial x}, \frac{\partial \overline{v'\chi_c'}}{\partial y}$$

No horizontal eddy flux divergence

Horizontally homogeneous flow

$$\overline{F_0} = \frac{1}{L^2}\int_0^L\int_0^L\int_0^h \overline{c_d}\,\overline{\frac{\partial \chi_c}{\partial t}}\,dxdydz$$

$$+\frac{1}{L^2}\int_0^L\int_0^L\int_0^h \left[\overline{uc_d}\,\cancel{\frac{\partial \overline{\chi_c}}{\partial x}} + \overline{vc_d}\,\cancel{\frac{\partial \overline{\chi_c}}{\partial y}} + \cancel{\overline{wc_d}}\,\frac{\partial \overline{\chi_c}}{\partial z}\right] dxdydz$$

$$+\frac{1}{L^2}\int_0^L\int_0^L\int_0^h \left[\cancel{\frac{\partial \overline{c_d}\,\overline{u'\chi_c'}}{\partial x}} + \cancel{\frac{\partial \overline{c_d}\,\overline{v'\chi_c'}}{\partial y}} + \frac{\partial \overline{c_d}\,\overline{w'\chi_c'}}{\partial z}\right] dxdydz$$

Horizontally homogeneous flow

Non-steady-state

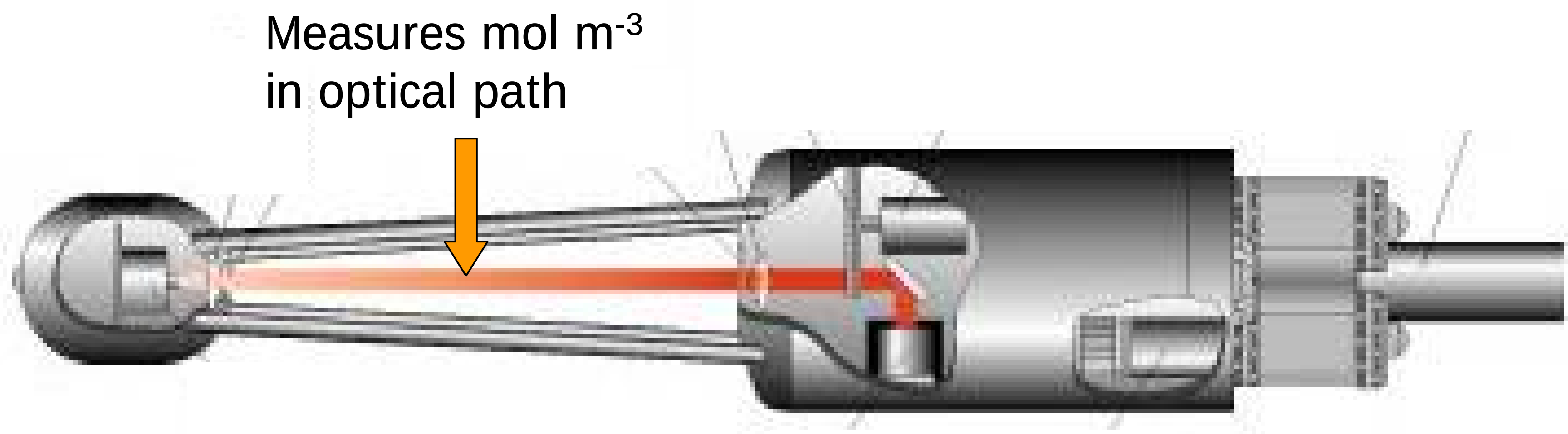
$$\overline{F_0} = \int_0^h \overline{c_d}\,\overline{\frac{\partial \chi_c}{\partial t}}\,dz + \overline{c_d}\,\overline{w'\chi_c'}$$

Steady-state eddy flux

$$\boxed{\overline{F_0} = \overline{c_d}\,\overline{w'\chi_c'}}$$

Time-averages at a point are assumed equal to space average – ergodic hypothesis

Concentrations measured, not mixing ratios



LI-7500 Open-path CO_2 and water vapor analyser

Webb, Pearman & Leuning (1980) theory (1)
Steady state, horiz. homogeneous flow

$$\overline{F_c} = \overline{w c_c} = \overline{w}\,\overline{c_c} + \overline{w' c_c'}$$ Trace gas flux

What is $\overline{w}$?

$$\overline{F_d} = 0 = \overline{w}\,\overline{c_d} + \overline{w' c_d'}$$ No net flux of dry air

$$\overline{w} = \frac{-\overline{w' c_d'}}{\overline{c_d}}$$ Need expression for c_d'

WPL theory (2)

$$\overline{w} = \frac{-\overline{w'c'_d}}{\overline{c_d}}$$

$$c'_d = c' - c'_v$$

$$c = \frac{p}{RT}, \qquad c' = -\overline{c}\left[\frac{T'}{\overline{T}} - \frac{p'}{\overline{p}}\right]$$

$$c'_d = -\overline{c}\left[\frac{T'}{\overline{T}} - \frac{p'}{\overline{p}}\right] - c'_v$$

$$\overline{w} = \frac{1}{\overline{c_d}}\left[\overline{w'c'_v} + \overline{c}\left(\frac{\overline{w'T'}}{\overline{T}} - \cancel{\frac{\overline{w'p'}}{\overline{p}}}\right)\right]$$

WPL theory (3)

- Open-path instruments

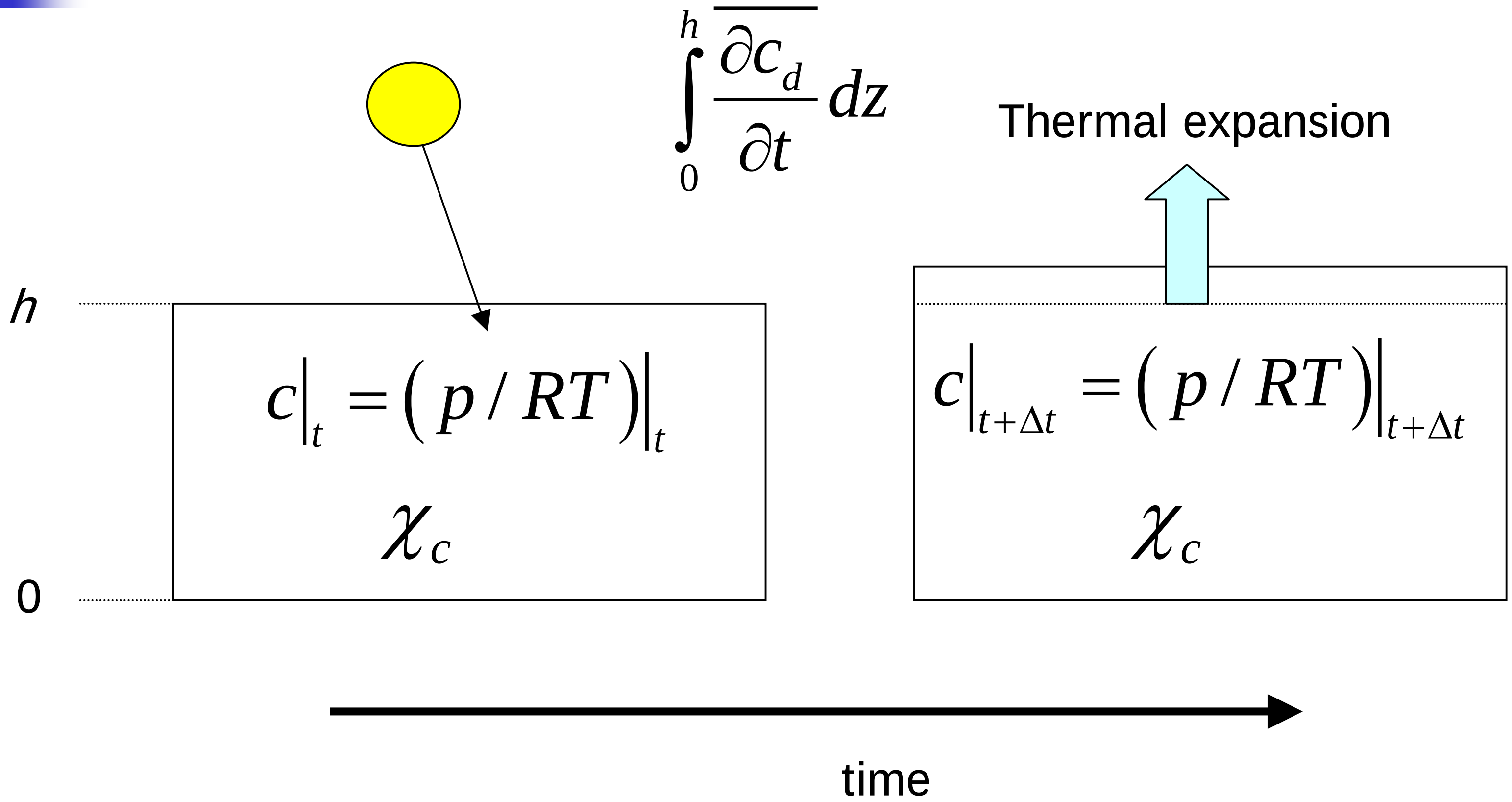
$$\overline{F_0} = \overline{c}_d \overline{w'\chi_c'} = \overline{w'c_c'} + \overline{\chi_c}\left[\overline{w'c_v'} + \overline{c}\frac{\overline{w'T'}}{\overline{\overline{T}}}\right]$$

Raw CO_2 flux

Water vapor flux

Heat flux

Non-steady state, horiz. homogeneous flow



WPL theory (4)

$$0 = \overline{F_d} = \int_0^h \overline{\frac{\partial c_d}{\partial t}}\, dz + \overline{w}\,\overline{c_d} + \overline{w' c_d'} \quad \text{No source/sink of dry air}$$

$$\overline{w} = \frac{-1}{\overline{c_d}\Delta t}\left[\int_0^h \left(c_d\big|_{t+\Delta t} - c_d\big|_t\right) dz\right] - \frac{\overline{w' c_d'}}{\overline{c_d}}$$

?

$$\overline{w} = -\frac{\overline{w' c_d'}}{\overline{c_d}}, \quad \text{original WPL}$$

WPL theory (5)

$$\overline{F_c} = \int_0^h \overline{\frac{\partial c_c}{\partial t}} dz + \overline{w}\,\overline{c_c} + \overline{w'c_c'}$$

Hand-out notes show that

$$\overline{F_c} = \frac{\overline{\chi_c}}{\Delta t}\left[\int_0^h (c_d|_{t+\Delta t} - c_d|_t) dz\right]$$

$$||$$

$$-\frac{\overline{\chi_c}}{\Delta t}\left[\int_0^h (c_d|_{t+\Delta t} - c_d|_t) dz\right] - \overline{\chi_c}\,\overline{w'c_d'} + \overline{w'c_c'}$$

$$\boxed{\overline{F_c} = \overline{w'c_c'} - \overline{\chi_c}\,\overline{w'c_d'} = \text{original WPL}}$$

How big is $\overline{w}$?

- In terms of latent & sensible heat fluxes

$$\overline{w} = 10^{-6}(0.54\lambda\overline{E} + 2.80\overline{H})$$

- < 3 mm s $^{-1}$
- Difficult to measure directly

Open path measurements – calculation sequence

$$1)\quad \overline{H} = \overline{\rho} c_p \overline{w' T'}$$

$$2)\quad \overline{E} = (1 + \overline{\chi}_v) \left[\overline{w' c_v'} + \frac{\overline{c}_v}{\overline{\overline{T}}} \frac{\overline{H}}{\overline{\overline{\rho}} c_p} \right]$$

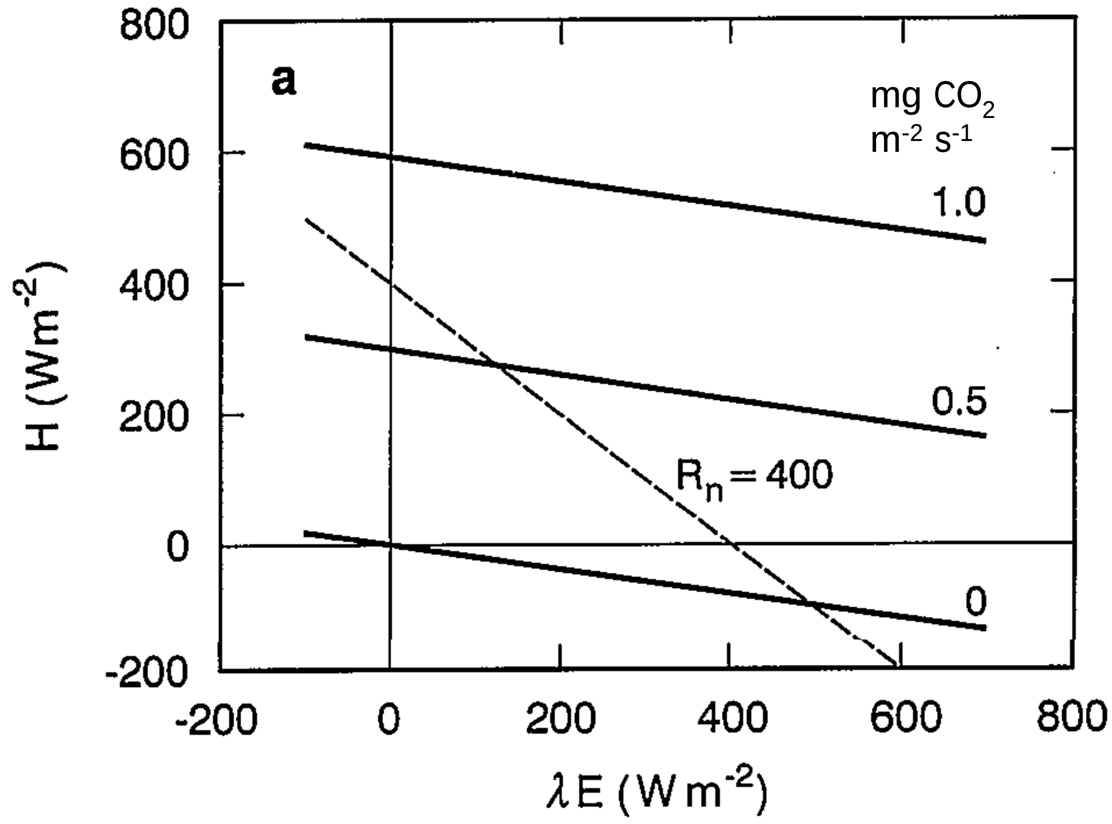
$$3)\quad \overline{F}_c = \overline{w' c_c'} + \overline{c}_c \left[\frac{\overline{E}}{\overline{\overline{c}}} + \frac{\overline{H}}{\overline{\overline{\rho}} c_p \overline{\overline{T}}} \right]$$

WPL corrections

1 mg CO_2 = 22.7 μmol CO_2

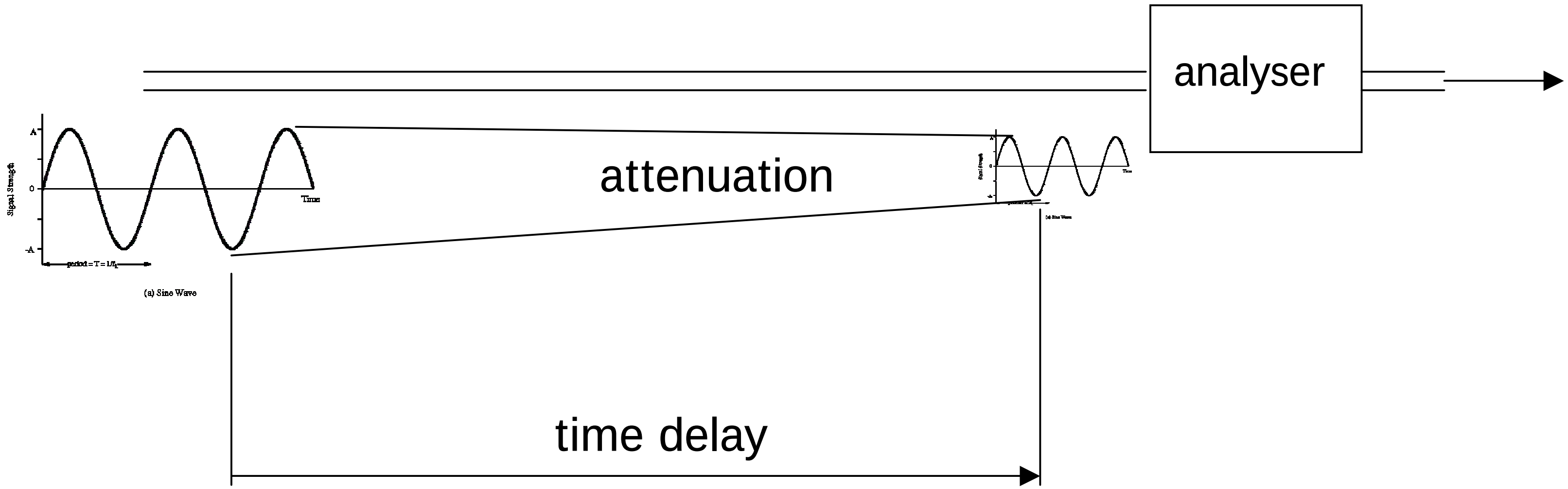
$$\overline{F}_c = \overline{w'c'_c} + \overline{c}_c \left[\frac{\overline{E}}{\overline{\overline{c}}} + \frac{\overline{H}}{\overline{\overline{\rho c_p}}\,\overline{\overline{T}}} \right]$$

Correction term

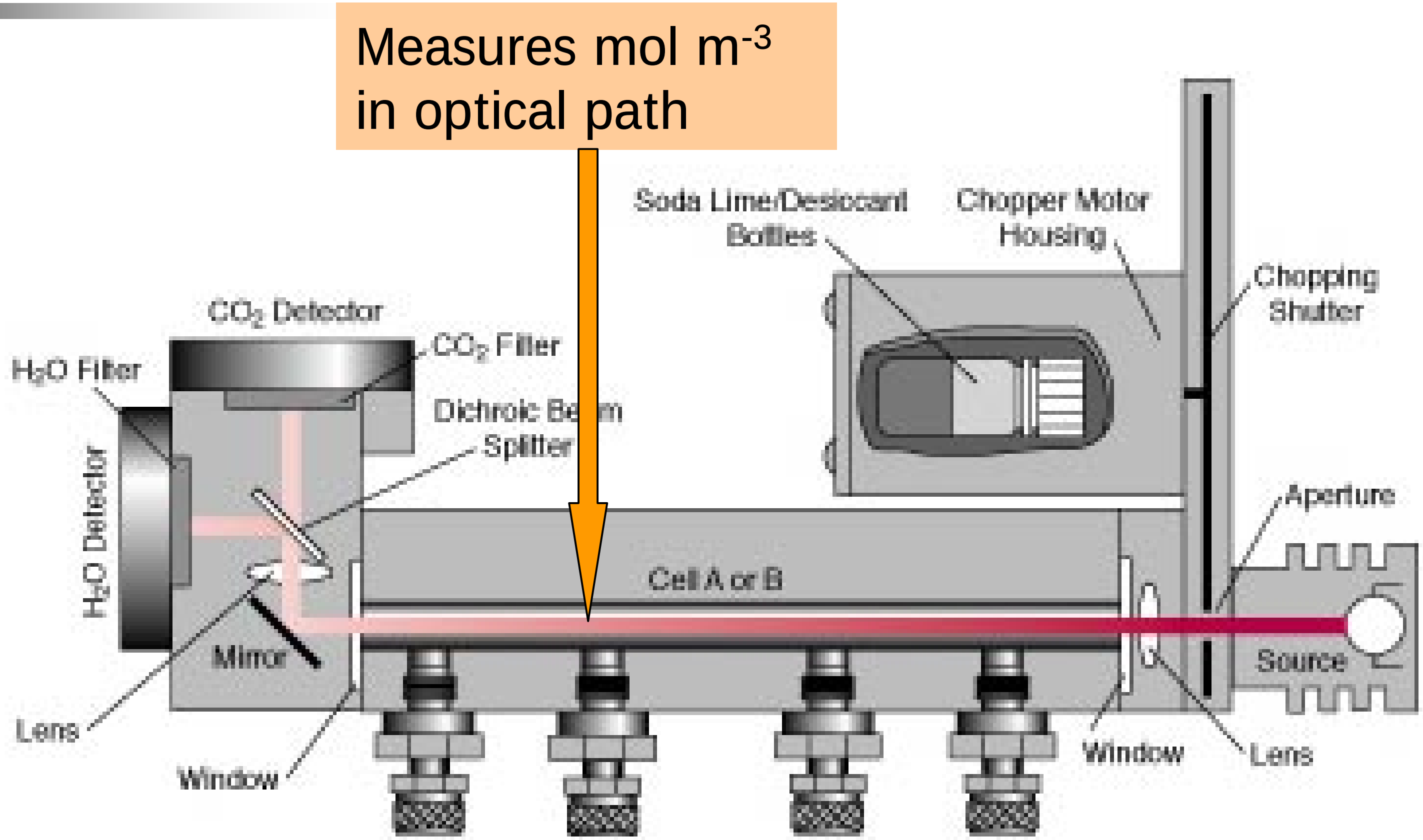


Leuning & Judd, 1996

Closed-path gas sampling



Licor 7000 Closed-path analyser



Closed-path analyser (1)

- *T* & *P* fluctuations must be removed for all frequencies $\geq 1/(2\pi t_{av})$
  - Air not dried

$$1)\quad \overline{E} = \frac{\overline{p}\,\overline{T}_i}{\overline{\overline{p_i}}\,\overline{\overline{T}}}(1+\overline{\chi}_v)\overline{w'c'_v}$$

$$2)\quad \overline{F}_c = \frac{\overline{p}\,\overline{T}_i}{\overline{\overline{p_i}}\,\overline{\overline{T}}}\left[\overline{w'c'_c} + \overline{\chi_c}\,\overline{w'c'_v}\right]$$

Closed-path analyser (2)

- Measure c_c, c_v, T & P simultaneously in gas analyser and calculate mixing ratio at sampling rate (10 Hz) used for eddy covariance

$$\chi_v = \frac{c_v}{P_i/(RT_i) - c_v}, \quad \chi_c = \frac{c_c}{P_i/(RT_i) - c_c}$$

- Must consider
  - Time-lag
  - Hi-frequency attenuation

Summary

- No net source/sink of dry air in CV
- Flux equation for trace gas is exact using mixing ratios relative to dry air for full, unsteady 3-D flows
- Use existing WPL corrections, even for 3-D flows